#### Week 7

## Ordered Dependent Variables

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October 4, 2012

#### Clarification

 Last week's notation might have been misleading for the heteroskedastic probit.

Var
$$[\varepsilon] = \sigma^2 = [e^{\gamma Z}]^2$$
  
So:

$$\sigma = e^{\gamma Z}$$

and the likelihood function is:

Ln 
$$L = \sum_{i=1}^{n} y_i \ln \Phi(\frac{X\beta}{e^{\gamma Z}}) + (1 - y_i) \ln \Phi(\frac{X\beta}{e^{\gamma Z}})$$

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- So far we have looked at continuous and dichotomous dependent variables.
- Continuous data have meaningful and constant distance between units and can theoretically be divided infinitely (e.g. money).

Now, let's move on to look at another type of dependent variable...ordinal variables.

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## Ordinality

- Ordinal data can be discrete or grouped continuous.
- Some data can be ordered but should not be.
- Some data are ordered in some situations but not others.
  - Long compares colors on the electromagnetic spectrum to that of buying cars of certain colors.
- Some treat ordered (discrete) variables as if they were continuous (e.g. use OLS).

## Ordinality

 Often seen in survey research with the seven point Likert scale.

- Strongly disagree
- Disagree
- Weakly disagree
- Neutral
- Weakly disagree
- Agree
- Strongly Agree

## Using OLS with ordinal data

- Some people use OLS with data with an ordinal outcome.
  - Do'h! OLS results, however, would be misleading.
- OLS assumes that the distance between categories is equal.
  - If this is the case, our estimates of  $\beta$  might be unbiased.
  - But the errors will be heteroskedastic and non-normal.
- If this assumption is not met, then the  $\beta$  estimates will be biased.

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## Latent variable approach

 Like logit and probit, we can motivate the ordered model via a latent variable approach.

 Suppose the unobservable latent variable y\* varies from  $-\infty$  to  $\infty$  and is mapped to an observed variable  $y_i$ .

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Then like in previous examples let

$$y^* = X\beta + \varepsilon$$

$$y_{i} = \begin{cases} 0, if \ y^{*} \leq 0 \\ 1, if \ 0 < y^{*} \leq \tau_{1} \\ 2, if \tau_{1} < y^{*} \leq \tau_{2} \end{cases}$$

$$\begin{cases} 3, if \ \tau_{2} < y^{*} \leq \tau_{3} \\ \dots \\ j, if \ \tau_{j-1} \leq y^{*} \end{cases}$$

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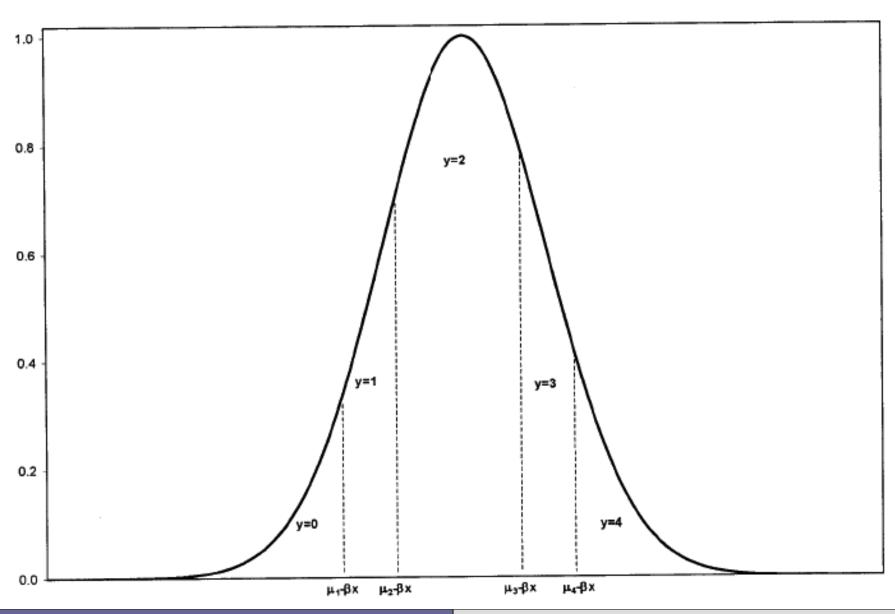
• Crucial to understanding the ordinal regression model (ORM) is  $\tau$ .

•  $\tau$  is the threshold or cut-point between categories.

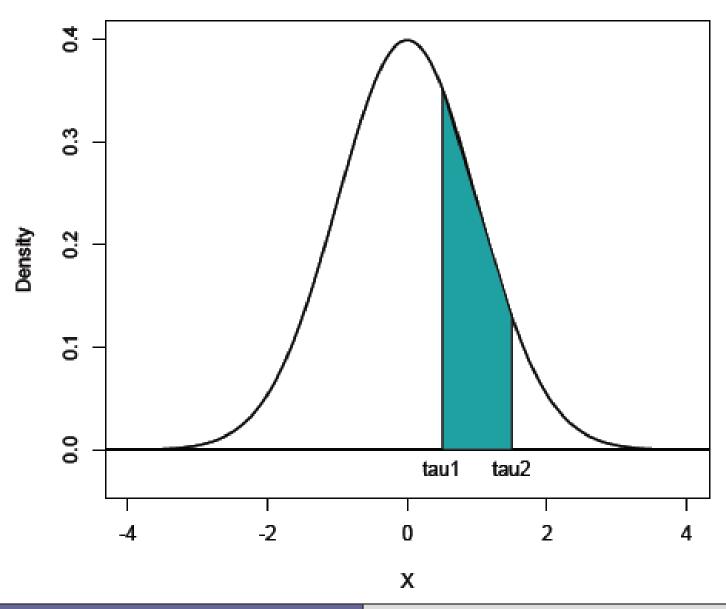
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# Probabilities and cut-points



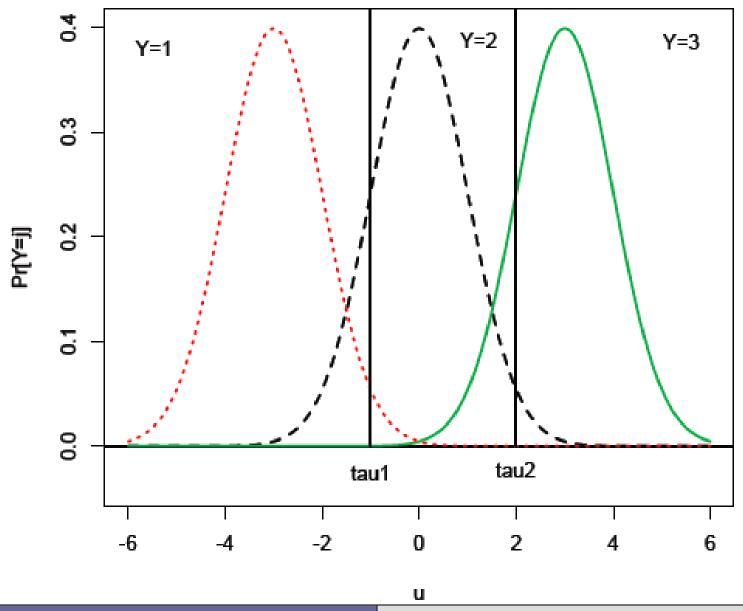
• Basically, as Long (1997: 121) puts it: "The probability that a random variable is between two values is the difference between the CDF evaluated at these values."



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## So you are basically subtracting one from the others.



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14

• The probability Y = j is given by:

$$Pr(Y = j/X) = F(\tau_j - X\beta) - F(\tau_{j-1} - X\beta)$$

• So the probability Y = 1 is the area between the two thresholds that delineate Y = 1.

$$Pr(Y = j/X) = F(\tau_j - X\beta) - F(-X\beta)$$

Or for probit models:

$$Pr(Y = j/X) = \Phi(\tau_j - X\beta) - \Phi(-X\beta)$$

- Given the fact that the ordered probit and logit are extensions of the binary model, estimation is relatively simple.
- Remember our old friend, the probit likelihood function:

$$L(\boldsymbol{\beta}|Y,X) = \prod_{i=1}^{n} [\Phi(X\boldsymbol{\beta})]^{y_i} [1 - \Phi(X\boldsymbol{\beta})]^{1-y_i}$$

and the log likelihood:

$$lnL = \sum_{i=1}^{n} y_i \ln \Phi(X\beta) + (1 - y_i) \ln \Phi(X\beta)$$

• We just substitute  $\tau_j$  in for  $y_i$ .

Therefore for the ordered probit the likelihood is:

$$L(\beta, \tau | Y, X) = \prod_{i=1}^{n} \prod_{j=1}^{m} [\Phi(\tau_{j} - \beta X) - \Phi(\tau_{j-1} - \beta X)]^{\tau_{ij}}$$

■ Then we log both sides to get the log likelihood.

$$\ln L(\beta, \tau \mid Y, X) = \sum_{i=1}^{n} \sum_{j=1}^{m} \tau_{ij} \ln [\Phi(\tau_{j} - \beta X) - \Phi(\tau_{j-1} - \beta X)]$$

## Heteroskedastic ordered probit?

- Theoretically, we could also model potential heteroskedasticity by parameterizing the variance  $\sigma^2$  by parameterizing it as  $e^{\gamma Z}$ .
- What examples can you think of that could use this?
- Unfortunately, Stata does not currently support an easy pre-baked heteroskedastic ordered probit.
- You could specify your own ML model and do it yourself!

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#### Identification

 Since y\* is unobserved, its mean and variance cannot be measured.

• We assume the variance to be 1 for ordered probit and  $\pi^2/3$  for logit.

 For identification we need to fix another variable to some arbitrary value.

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#### Identification

 As Long (1997: 123) writes, there are two frequently used assumptions

- Assume  $\tau_1 = 0$
- Assume  $\alpha = 0$
- Different software packages use one or the other.
  - Stata assumes  $\alpha = 0$ .
- This choice affects the interpretation of the output.
- The predictions, however, stay the same.

#### Let's look at some real data

■ Englehart, Neil A. 2009. "State Capacity, State Failure, and Human Rights." *Journal of Peace Research* 46(2): 163-180.

As close to Cingranelli and Filippov (2010) that I could find.

 This article looks at how state capacity affects observed levels of human rights.

## DV= Political Terror Scale

Level	Description
1	Countries under a secure rule of law, people are not imprisoned for their views, and torture is rare or exceptional. Political murders are extremely rare.
2	There is a limited amount of imprisonment for nonviolent political activity. However, few persons are affected, torture and beatings are exceptional. Political murder is rare.
3	There is extensive political imprisonment, or a recent history of such imprisonment. Execution or other political murders and brutality may be common. Unlimited detention, with or without a trial, for political views is accepted.
4	Civil and political rights violations have expanded to large numbers of the population. Murders, disappearances, and torture are a common part of life. In spite of its generality, on this level terror affects those who interest themselves in politics or ideas.
5	Terror has expanded to the whole population. The leaders of these societies place no limits on the means or thoroughness with which they pursue personal or ideological goals.

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• Therefore, when estimating what level of political terror a state will have we use the following:

$$Pr(Y = 1) = \Phi(\tau_1 - X\beta)$$
  
 $Pr(Y = 2) = \Phi(\tau_2 - X\beta) - \Phi(\tau_1 - X\beta)$   
 $Pr(Y = 3) = \Phi(\tau_3 - X\beta) - \Phi(\tau_2 - X\beta)$   
 $Pr(Y = 4) = \Phi(\tau_4 - X\beta) - \Phi(\tau_3 - X\beta)$   
 $Pr(Y = 5) = 1 - \Phi(\tau_5 - X\beta)$ 

 Englehart inverts the PTS scale so that higher scores represent better human rights conditions.

- Independent variables capturing state capacity:
  - Political Risk Service's Law and Order measure
    - Strength and impartiality of legal system/degree to which law is respected and order maintained.
  - Transparency International's Corruption Perceptions Index (CPI)
  - Taxes as a proportion of GDP

## Summary data

. sum law\_order\_prs cpi05 taxgdp\_best cwardead iwardead polity2 gdppccus06ln poptot06ln igoprop iccpropt1 ptss2in1

Variable	Obs	Mean	Std. Dev.	Min	Max
law order ~w	2552	3.633138	1.549606	0	6
cpi05	1266	4.926619	2.546021	0	10
taxgdp best	4139	.1912724	.0994063	.0003458	.8837426
cwardead	7227	503.8608	3329.971	0	80000
iwardead	7227	3087.54	22874.13	0	323914
polity2	6008	1709387	7.5282	-10	10
gdppccus06ln	5421	7.430181	1.560977	4.034598	10.87698
poptot06ln	6885	15.51374	1.886037	9.888374	20.98267
igoprop	6282	1.000449	.4255889	.0200336	2.768744
iccpropt1	7198	.2399278	.4270688	0	1
ptss2in1	3940	3.675127	1.1674	1	5

■ I replicate the first three columns of Table 1 (Englehart 2009: 172) without including year dummies, then one model with all three IVs.

7.222497

15.59198

```
. ologit ptss2inv law order prs cpi05 taxgdp best cwardead iwardead polity2 gdp
> pccus06ln poptot06ln igoprop iccpropt1 ptss2in1, robust cluster(banks) nolog
Ordered logistic regression
                                                   Number of obs
                                                                            628
                                                   Wald chi2(11)
                                                                         355.09
                                                   Prob > chi2
                                                                         0.0000
Log pseudolikelihood = -386.39185
                                                   Pseudo R2
                                                                         0.5552
                                  (Std. Err. adjusted for 90 clusters in banks)
                             Robust
    ptss2inv |
                            Std. Err.
                                                           [95% Conf. Interval]
law order ~w |
                  .297825
                            .0886246
                                         3.36
                                                0.001
                                                          .1241239
                                                                       .4715262
       cpi05 |
                 .1312412
                            .0915943
                                         1.43
                                               0.152
                                                          -.0482803
                                                                       .3107627
                2.776397
                             1.48146
                                                0.061
                                                          -.1272105
                                                                       5.680004
 taxgdp best |
                                        1.87
    cwardead | -.0000168
                            .0000815
                                        -0.21
                                                0.836
                                                          -.0001766
                                                                       .0001429
    iwardead | -.0000223
                           6.56e-06
                                        -3.40
                                                0.001
                                                          -.0000351
                                                                      -9.42e-06
    polity2 |
                .0128612
                             .021753
                                         0.59
                                                0.554
                                                          -.0297739
                                                                       .0554963
gdppccus06ln |
               .0712448
                             .123099
                                         0.58
                                                0.563
                                                          -.1700249
                                                                       .3125144
 poptot06ln | -.3215179
                            .1250645
                                         -2.57
                                                0.010
                                                          -.5666398
                                                                      -.0763961
     igoprop |
                1.277837
                            .5276149
                                         2.42
                                                0.015
                                                          .243731
                                                                       2.311943
   iccpropt1 |
                 .0107571
                             .268408
                                         0.04
                                                0.968
                                                          -.5153129
                                                                       .5368271
                 2.616011
                             .2158452
                                        12.12
                                                0.000
                                                           2.192962
    ptss2in1 |
                                                                       3.039059
       /cut1 | -.0540404
                            2.289588
                                                           -4.54155
                                                                       4.433469
       /cut2 |
                3.954703
                            2.145507
                                                          -.2504131
                                                                       8.15982
       /cut3 |
                 7.712964
                            2.121039
                                                           3.555804
                                                                       11.87012
```

2.13511 MLE

/cut4 |

11.40724

 Although unsurprisingly these three variables are correlated above .5.

0.5593

cpi05 | 0.7260

taxgdp best

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1.0000

0.5873

1.0000

# Differences between OLS and oprobit

. estimates table OLS oprobit, b(%9.3f) t label varwidth(30) equations(1:1)

Variable	OLS	oprobit
#1 law and order	   0.094   7.05	0.212 7.50
prio civil war deaths	-0.000 -2.48	-0.000 -2.22
prio international war deaths	-0.000 -0.33	-0.000 -0.97
revised polity score	0.007 2.86	0.019 3.36
gdppccus06ln	0.029 1.67	0.080 2.24
poptot06ln	-0.083   -5.99	-0.181 -6.11
igoprop	0.100 1.57	0.391 2.58
iccpropt1	-0.030 -0.85	-0.080 -0.98
ptss2in1	0.703	1.321 20.18
Constant	1.736   6.42	
cut1 Constant	 	-0.149 -0.28
cut2 Constant	 	1.455 2.64
cut3 Constant	 	3.293 5.75
cut4 Constant	 	5.258 8.97
		legend: b/t

# Comparing ologit and oprobit

. estimates table ologit oprobit, b(%9.3f) t label varwidth(30)

Variable	   ologit	oprobit
ptss2inv	+ ı	
-	0.374 7.29	0.212 7.50
prio civil war deaths	-0.000   -1.68	-0.000 -2.22
prio international war deaths	-0.000 -0.85	-0.000 -0.97
revised polity score	0.034 3.37	0.019 3.36
gdppccus06ln	0.140 2.21	0.080 2.24
poptot06ln	-0.325   -6.08	-0.181 -6.11
igoprop	0.654 2.45	0.391 2.58
iccpropt1	-0.121 -0.86	-0.080 -0.98
ptss2in1	2.412   21.91	1.321 20.18
cut1 Constant	   -0.379   -0.39	-0.149 -0.28
cut2 Constant	   2.627   2.68	1.455 2.64
cut3 Constant	5.986 5.90	3.293 5.75
cut4 Constant	   9.542   9.24	5.258 8.97
		logond: h/t

legend: b/t

#### Can also run Wald tests of coefficients

```
. * Wald test
. oprobit ptss2inv law order prs new cwardead iwardead polity2 gdppccus06ln ///
> poptot06ln igoprop iccpropt1 ptss2in1, robust cluster(banks)
Iteration 0: log pseudolikelihood = -2701.9243
Iteration 1: log pseudolikelihood = -1487.3775
Iteration 2: log pseudolikelihood = -1404.0518
Iteration 3: log pseudolikelihood = -1402.6146
Iteration 4: log pseudolikelihood = -1402.6129
Iteration 5: log pseudolikelihood = -1402.6129
Ordered probit regression
                                         Number of obs =
                                                           1801
                                         Wald chi2(9) =
                                                           795.83
                                       Prob > chi2 =
                                                           0.0000
Log pseudolikelihood = -1402.6129
                                       Pseudo R2 =
                                                           0.4809
                          (Std. Err. adjusted for 125 clusters in banks)
   ptss2inv | Coef. Std. Err. z P>|z| [95% Conf. Interval]
law order ~w | .2120258 .0282618 7.50 0.000 .1566337
                                                        .2674178
   cwardead | -.000061 .0000274 -2.22 0.026 -.0001147 -7.22e-06
   polity2 | .0193145 .0057418 3.36 0.001 .0080608 .0305683
gdppccus06ln | .08046 .0359567 2.24 0.025 .0099861 .1509338
 poptot06ln | -.1812513 .0296876 -6.11 0.000 -.2394378 -.1230647
   igoprop | .3910244 .1516026 2.58 0.010 .0938887 .6881601
  iccpropt1 | -.0799299 .0813197 -0.98 0.326 -.2393136 .0794538
   ptss2in1 | 1.321426 .0654844 20.18 0.000 1.193079 1.449773
     /cut1 | -.1489925 .5408084
                                               -1.208958
                                                        .9109725
                                              .3747777 2.535738
     /cut2 | 1.455258 .5512756
     /cut3 | 3.292576 .5722217
                                              2.171042
                                                        4.41411
     /cut4 | 5.258085 .5860828
                                               4.109384 6.406786
. test law order prs
 (1) [ptss2inv]law order prs new = 0
         chi2(1) = 56.28
       Prob > chi2 = 0.0000
```

#### And LR tests

```
* LR test
** Unconstrained ***
.oprobit ptss2inv law order prs new cwardead iwardead polity2
gdppccus06ln ///
poptot06ln igoprop iccpropt1 ptss2in1
 estimates store fullmodel
** Constrained **
oprobit ptss2inv iwardead polity2 gdppccus06ln ///
 poptot06ln igoprop iccpropt1 ptss2in1 if law ~=.
estimates store constmodel
. lrtest fullmodel constmodel
```

Likelihood-ratio test

(Assumption: constmodel nested in fullmodel)

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LR chi2(2) = 80.95

0.0000

31

Prob > chi2 =

#### Can also run fitstat for more measures of fit

#### . fitstat

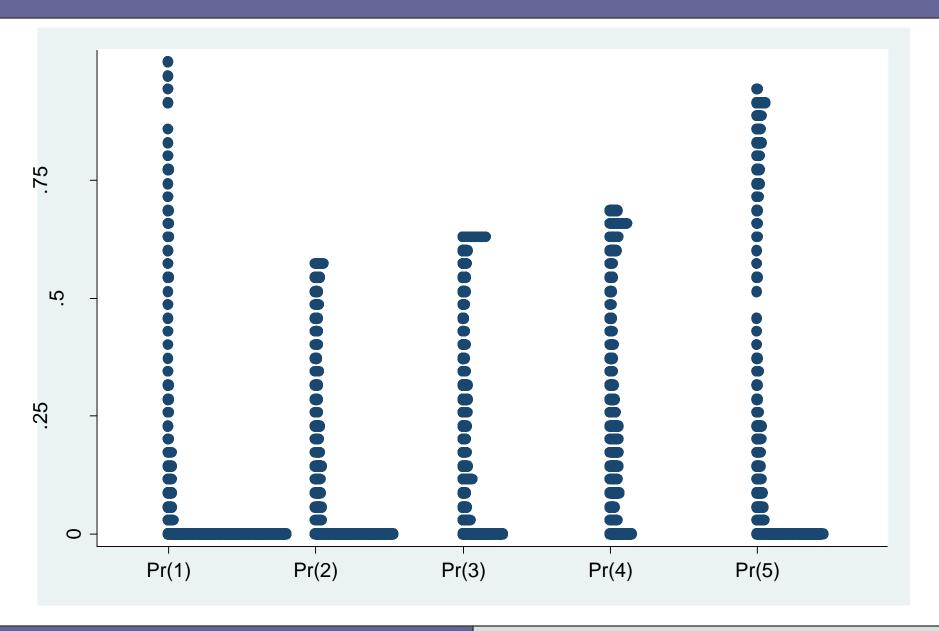
Measures of Fit for oprobit of ptss2inv

```
Log-Lik Intercept Only:
                           -2701.924
                                      Log-Lik Full Model:
                                                                  -1402.613
D(1788):
                            2805.226
                                                                   2598.623
                                      LR(9):
                                       Prob > LR:
                                                                      0.000
                             0.481 McFadden's Adj R2:
McFadden's R2:
                                                                      0.476
ML (Cox-Snell) R2:
                              0.764
                                      Cragg-Uhler(Nagelkerke) R2:
                                                                      0.804
McKelvey & Zavoina's R2:
                              0.815
Variance of y*:
                               5.418 Variance of error:
                                                                      1.000
Count R2:
                               0.713 Adj Count R2:
                                                                      0.596
                               1.572 AIC*n:
                                                                   2831.226
AIC:
                          -10597.796 BIC':
                                                                  -2531.158
BIC:
                            2902.675 AIC used by Stata:
                                                                   2831.226
BIC used by Stata:
```

end of do-file

32

# Predicted probabilities



### Predicted probabilities

# Individual predicted probabilities

	Political Terror Scale				
	1	2	3	4	5
Mean values (no ICC protocol)	0.0001	0.0184	0.3837	0.5549	0.0429
With ICC ratification	0.0002	0.0223	0.4109	0.5305	0.0361
Autocratic state (Polity=-6; no ICC)	0.0002	0.0269	0.4382	0.5045	0.0302
Democratic state (Polity=6; no ICC)	0.0001	0.0155	0.3594	0.5753	0.0498

#### Stata commands

```
oprobit ptss2inv law_order_prs_new cwardead iwardead polity2
gdppccus06ln poptot06ln igoprop iccpropt1 ptss2in1, robust
cluster(banks)

prvalue, x(iccpropt1=0) rest(mean)
prvalue, x(iccpropt1=1) rest(mean)
prvalue, x(polity2=-6 iccpropt1=0) rest(mean) brief
prvalue, x(polity2=6 iccpropt1=0) rest(mean) brief
```

#### Continuous probabilities

• Like last week, we are interested in trying to understand how an IV affects our DV.

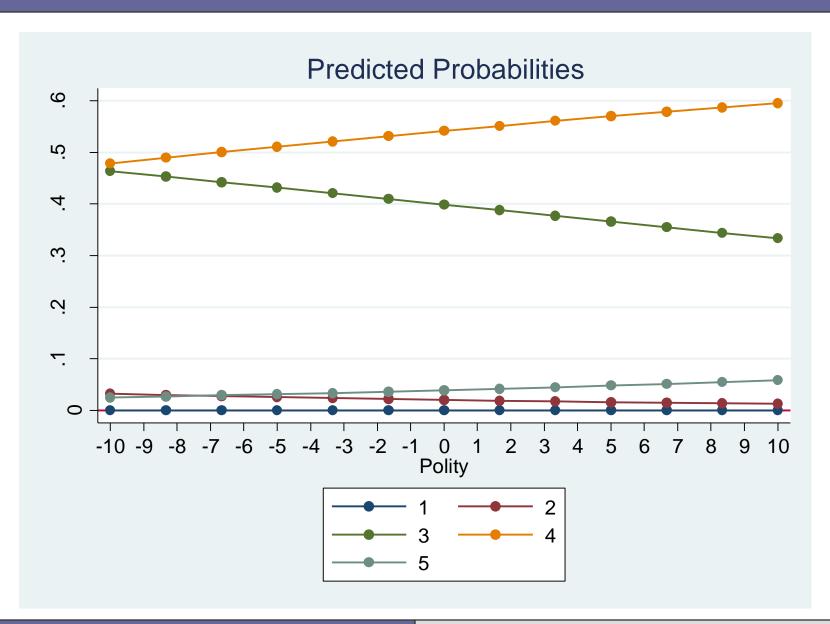
This is a bit more complicated than with logit or probit, but it is possible.

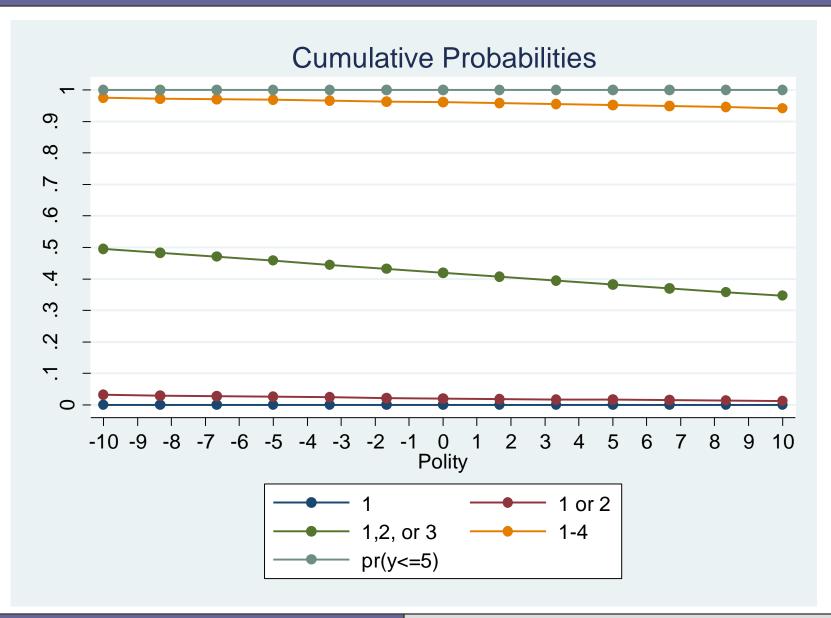
# Graphing

```
// graphing predicted probabilities with -prgen-
prgen polity2, from (-10) to (10) generate (nope) x (iccpropt1=0) ncases (13)
desc nope*
label var nopep1 "1"
label var nopep2 "2"
label var nopep3 "3"
label var nopep4 "4"
label var nopep5 "5"
label var nopes1 "1"
label var nopes2 "1 or 2"
label var nopes3 "1,2, or 3"
label var nopes4 "1-4"
* step 1: graph predicted probabilities
graph twoway connected nopep1 nopep2 nopep3 nopep4 nopep5 nopex, ///
     title ("Panel A: Predicted Probabilities") ///
     xtitle("Polity") xlabel(-10(1)10) ylabel(0(.25).50) ///
      ylabel(0(.1).6) yline(0) ///
      ytitle("Pr(Y = j)") name(tmp1, replace)
* step 2: graph cumulative probabilities
graph twoway connected nopes1 nopes2 nopes3 nopes4 nopes5 nopex, ///
    title ("Panel B: Cumulative Probabilities") ///
    xtitle("Polity") xlabel(-10(1)10) ylabel(0(.25).50) ///
    yscale(noline) ylabel(0(.1)1) name(tmp2, replace) ///
    ytitle("Pr(Y = j)")
```

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38





- There are a number of other means of interpretation.
  - Marginal effects
  - Discrete change
  - Odds ratios

 See Long (1997: 127-140) and Long and Freese (2006: 202-220).

• However, before we get too carried away with interpreting our results, we need to think about a fundamental assumption of ORM.

■ This is that an independent variable (Polity or whatever) has the same effect on all categories of the dependent variable.

# Parallel regression assumption

$$\frac{\delta \Pr(Y_i = j)}{\delta X} = \frac{\delta \Pr(Y_i = j')}{\delta X} \ \forall \ j \neq j'$$

Also known as the "proportional odds" assumption

• More simply, for the data above ORM assumes that:

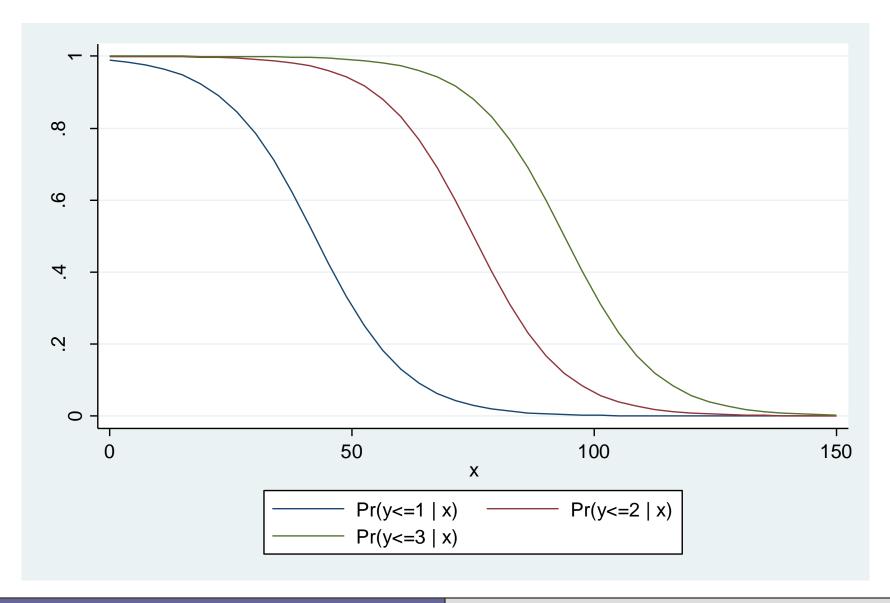
$$\beta_{Polity for y=1} = \beta_{Polity for y=2} = \beta_{Polity for y=3} = \beta_{Polity for y=4} = \beta_{Polity for y=5}$$

As you can see with the colors example, sometimes they are ordered in some instances and not others.

 Or the Likert scale can be imposing a different ordering than you are theoretically interested in.

• E.g. the difference between <u>intensity</u> of opinion and <u>direction</u> of opinion.

# We constrain the slopes to be the same.

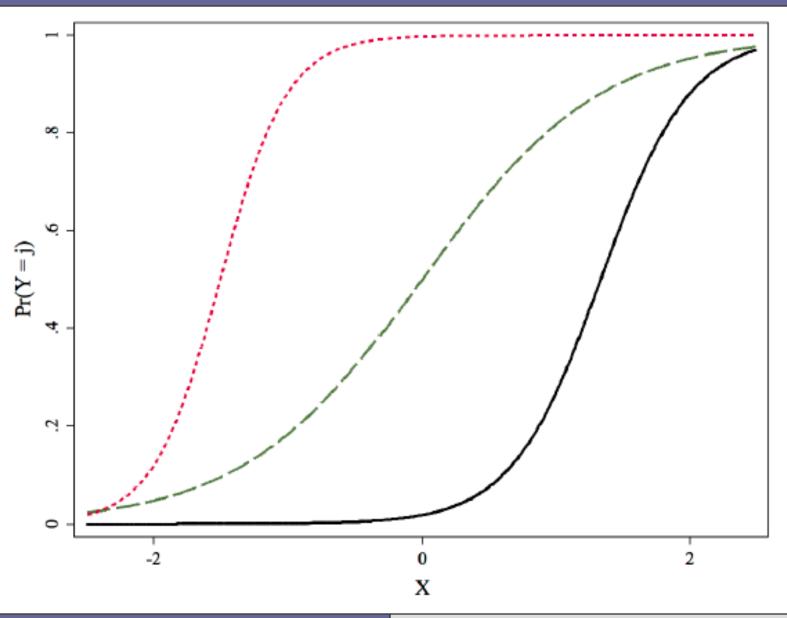


# Relaxing the Parallel Regression Assumption

$$\frac{\delta \Pr(Y_i = j)}{\delta X} \neq \frac{\delta \Pr(Y_i = j')}{\delta X} \ \forall \ j \ \neq j'$$

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## Which allows the slopes to vary across categories.



- We can find out if our data violate the parallel regression assumption in several ways.
  - LR Score test
  - Wald test

#### LR Score test

- . omodel logit ptss2inv law order prs new cwardead iwardead polity2 gdppccus06ln
- > poptot06ln igoprop iccpropt1 ptss2in1

```
Ordered logit estimates
                                     Number of obs = 1801
                                     LR chi2(9) = 2634.73
                                     Prob > chi2 = 0.0000
                                    Pseudo R2 = 0.4876
Log likelihood = -1384.5615
   ptss2inv | Coef. Std. Err. z P>|z| [95% Conf. Interval]
law order ~w | .3744251 .0481613 7.77 0.000 .2800307 .4688195
  cwardead | -.0001238 .0000357 -3.47 0.001 -.0001938 -.0000538
  polity2 | .0337874 .0086336 3.91 0.000 .0168659 .0507089
gdppccus06ln | .1402367 .0505103 2.78 0.005 .0412383 .2392352
 poptot06ln | -.3250962 .046525 -6.99 0.000 -.4162835 -.2339089
   igoprop | .6535872 .2332295 2.80 0.005 .1964657 1.110709
  iccpropt1 | -.1212637 .1191453 -1.02 0.309 -.3547843 .1122568
  ptss2in1 | 2.412143
                    .0846339 28.50 0.000
                                        2.246264
                                                   2.578023
    cut1 | -.3786732 .8287469
                                 (Ancillary parameters)
    cut2 | 2.627115 .8327884
    _cut3 | 5.98563 .8291999
     cut4 | 9.542013 .8547224
```

Approximate likelihood-ratio test of proportionality of odds across response categories:

```
chi2(27) = 127.97

Prob > chi2 = 0.0000
```

#### Wald Test

. brant, detail

Estimated coefficients from j-1 binary regressions

Brant Test of Parallel Regression Assumption

Variable	chi2	p>chi2	df
All	131.69	0.000	27
law_order_~w cwardead iwardead polity2 gdppccus06ln poptot06ln igoprop iccpropt1 ptss2in1	8.46   36.85   4.36   14.10   8.46   10.48   5.15   4.88   3.94	0.037 0.000 0.225 0.003 0.037 0.015 0.161 0.181 0.268	3 3 3 3 3 3 3 3

A significant test statistic provides evidence that the parallel regression assumption has been violated.

• As you can see from both tests, these data violate the parallel regression assumption, and Englehart should have looked to other methods to estimate his model with.

 We will learn more about such models in two weeks.

- As we have seen the ordinal regression model has some assumptions that our data can easily violate.
- The most notable is the parallel regression assumption.
- This constrains the effect of our IVs to be the same over the range of Y outcomes.
- If this assumption is violated and ORM is used, the results can be biased at best, and nonsensical at worst.

 Let's turn to another example of ordered probit...Cingranelli and Filippov (2010).

# Cingranelli and Fillipov 2010

 Replication data unfortunately not available online.

 Instead of PTS, they use the Cingranelli-Richards (CIRI) human rights data.

## Physical Integrity Rights Index

■ This is an additive index constructed from the Torture, Extrajudicial Killing, Political Imprisonment, and Disappearance indicators. It ranges from 0 (no government respect for these four rights) to 8 (full government respect for these four rights).

Thoughts on this article?

• Their methods?

■ Their interpretation of the models?

• Let's discuss with the remaining time the types of interpretation we learned about last week...

 For example, predicted probabilities of our latent DV given a range of an independent variable of interest.